Calculus AB

4-4

(Day 3)

Second Fundamental Theorem of Calculus

The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then for every x in the interval,

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\left[F(x)-F(a)\right] = F(x) - 0$$

integrals and derivatives cancel each other (inverses.)

Find F as a function of x and evaluate it at $x = 2$, $x = 5$, and $x = 8$. (pg 294)

68) $F(x) = \int_2^x (t^3 + 2t - 2)\,dt$

$$= \frac{1}{4}t^4 + t^2 - 2t\Big|_2^x = \left[\frac{1}{4}x^4 + x^2 - 2x\right] - \left[\frac{1}{4}2^4 + 2^2 - 2\cdot 2\right]$$

$$\frac{1}{4}x^4 + x^2 - 2x - 4$$

$$F(2) = \frac{1}{4}(2)^4 + 2^2 - 2(2) - 4 = 0$$

$$F(5) = \frac{1}{4}(5)^4 + 5^2 - 2(5) - 4 = \boxed{167.25}$$

Use the same process to find f(8). It was skipped to save time.

(a) Integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).

76) $F(x) = \int_0^x t(t^2 + 1)\,dt = \int_0^x (t^3 + t)\,dt$

$$F(x) = \frac{1}{4}t^4 + \frac{1}{2}t^2\Big|_0^x = \frac{1}{4}x^4 + \frac{1}{2}x^2 - 0$$

$$F(x) = \frac{1}{4}x^4 + \frac{1}{2}x^2$$

b) $F'(x) = x^3 + x = x(x^2+1) = f(x)$

Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

82) $F(x) = \int_1^x \frac{t^2}{t^2+1}\,dt$

$$F'(x) = \frac{x^2}{x^2+1}$$

Find $F'(x)$.

90) $F(x) = \int_2^{x^2} \frac{1}{t^3}\,dt$

Chain Rule

$$F'(x) = \frac{2x}{(x^2)^3} = \frac{2x}{x^6} = \boxed{\frac{2}{x^5}}$$

Chain Rule

$$\frac{d}{dx}\int_a^{h(x)} g(t)\,dt = g(h(x))\,h'(x)$$

Assignment:
Pg. 294
67-91 odd